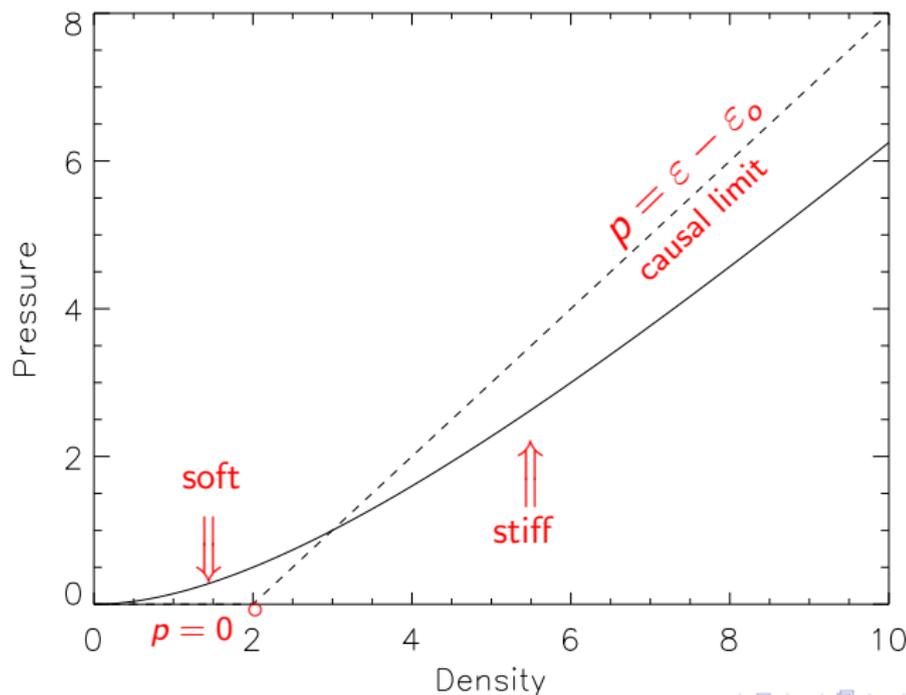


Extreme Properties of Neutron Stars

- ▶ The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ϵ_0 is the only
EOS parameter

The TOV
solutions scale
with ϵ_0

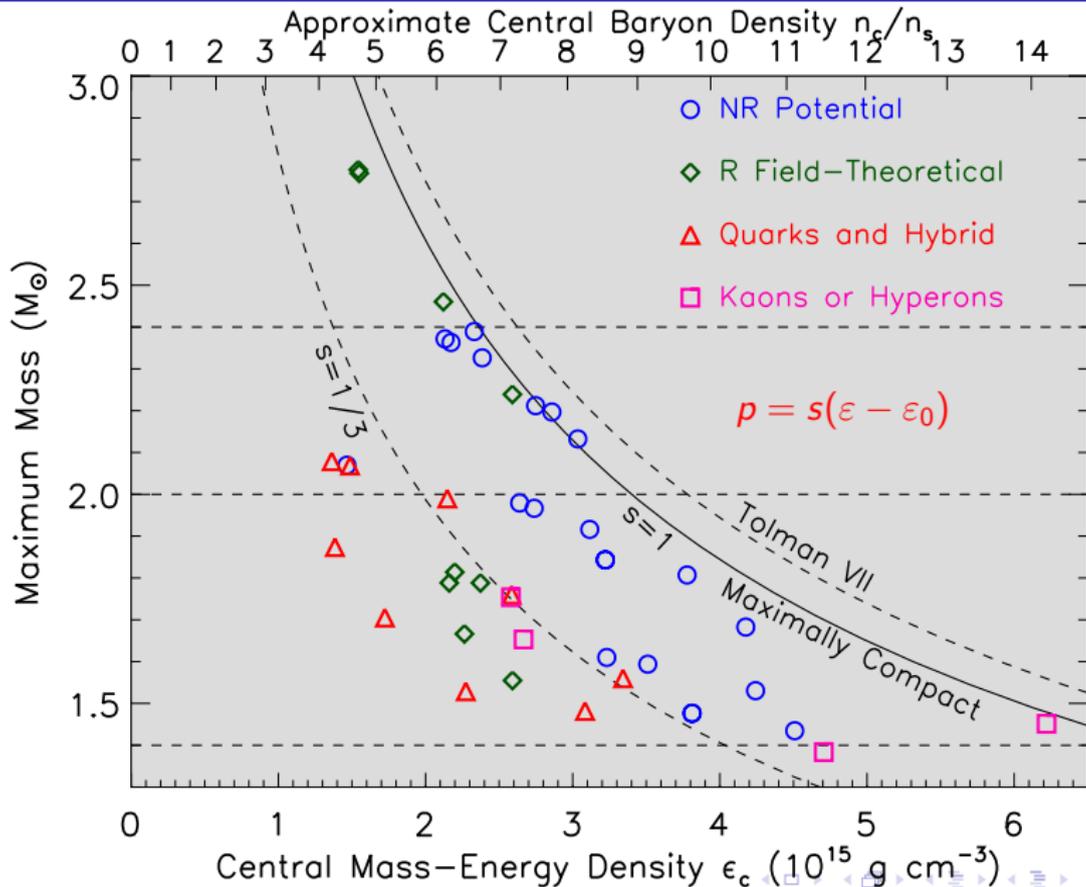
Extreme Properties of Neutron Stars

- ▶ $M_{max} = 4.1 (\epsilon_s/\epsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- ▶ $M_{B,max} = 5.41 (m_B c^2/\mu_o)(\epsilon_s/\epsilon_0)^{1/2} M_\odot$
- ▶ $R_{min} = 2.82 GM/c^2 = 4.3 (M/M_\odot) \text{ km}$
- ▶ $\mu_{B,max} = 2.09 \text{ GeV}$
- ▶ $\epsilon_{c,max} = 3.034 \epsilon_0 \simeq 51 (M_\odot/M_{largest})^2 \epsilon_s$
- ▶ $p_{c,max} = 2.034 \epsilon_0 \simeq 34 (M_\odot/M_{largest})^2 \epsilon_s$
- ▶ $n_{B,max} \simeq 38 (M_\odot/M_{largest})^2 n_s$
- ▶ $BE_{max} = 0.34 M$
- ▶ $P_{min} = 0.74 (M_\odot/M_{sph})^{1/2} (R_{sph}/10 \text{ km})^{3/2} \text{ ms}$
 $= 0.20 (M_{sph,max}/M_\odot) \text{ ms}$

A phenomenological limit for hadronic matter (Lattimer & Prakash 2004)

- ▶ $P_{min} \simeq 1.00 (M_\odot/M_{sph})^{1/2} (R_{sph}/10 \text{ km})^{3/2} \text{ ms}$
 $= 0.27 (M_{sph,max}/M_\odot) \text{ ms}$

Maximum Energy Density in Neutron Stars



Mass-Radius Diagram and Theoretical Constraints

GR:

$$R > 2GM/c^2$$

$P < \infty$:

$$R > (9/4)GM/c^2$$

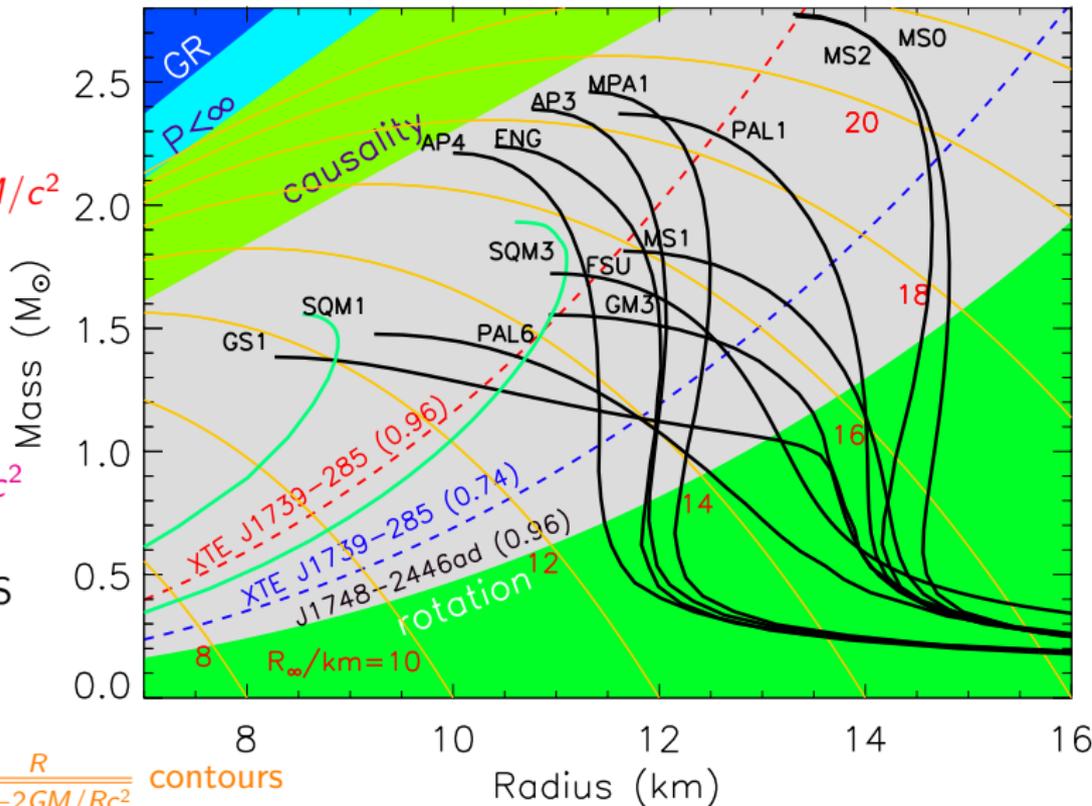
causality:

$$R \gtrsim 2.9GM/c^2$$

— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



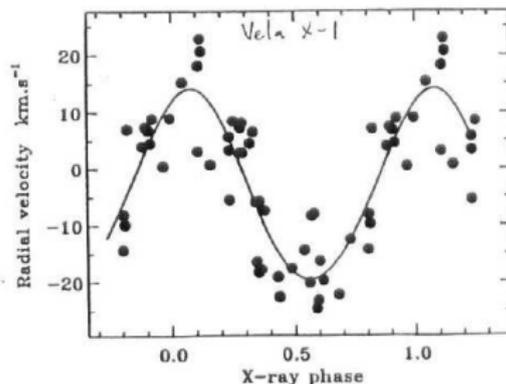
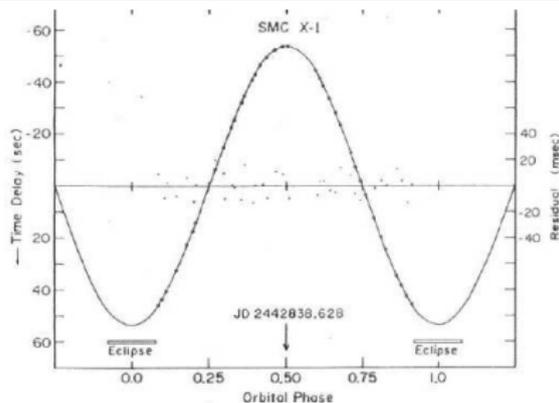
Mass Measurements In X-Ray Binaries

Mass function

$$\begin{aligned}f(M_1) &= \frac{P(v_2 \sin i)^3}{2\pi G} \\ &= \frac{(M_1 \sin i)^3}{(M_1 + M_2)^2} \\ &> M_1\end{aligned}$$

$$\begin{aligned}f(M_2) &= \frac{P(v_1 \sin i)^3}{2\pi G} \\ &= \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} \\ &> M_2\end{aligned}$$

In an X-ray binary, v_{optical} has the largest uncertainties. In some cases, $\sin i \sim 1$ if eclipses are observed. If eclipses are not observed, limits to i can be made based on the estimated radius of the optical star.



Pulsar Mass Measurements

Mass function for pulsar precisely obtained.

It is also possible in some cases to obtain the rate of periastron advance and the Einstein gravitational redshift + time dilation term:

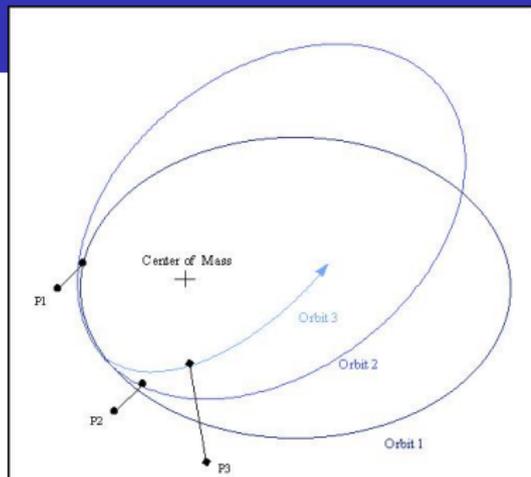
$$\dot{\omega} = 3(2\pi/P)^{5/3} (GM/c^2)^{2/3} / (1 - e^2)$$

$$\gamma = (P/2\pi)^{1/3} e M_2 (2M_2 + M_1) (G/M^2 c^2)^{2/3}$$

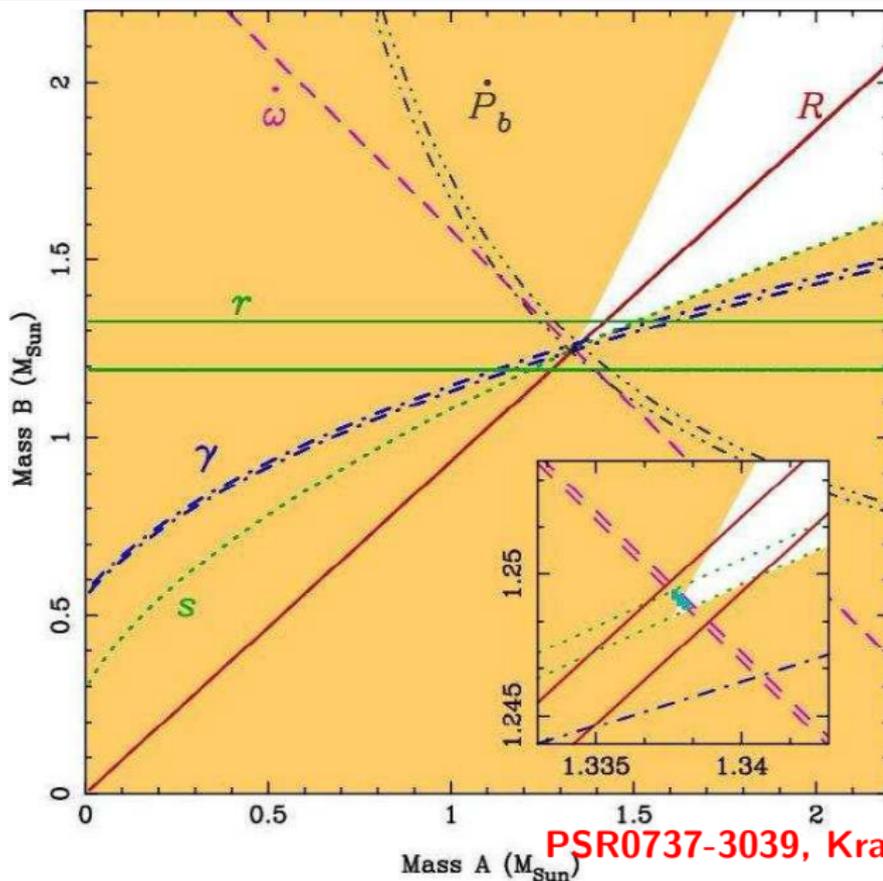
Gravitational radiation leads to orbit decay:

$$\dot{P} = -\frac{192\pi}{5c^5} \left(\frac{2\pi G}{P}\right)^{5/3} (1 - e^2)^{-7/2} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \frac{M_1 M_2}{M^{1/2}}$$

In some cases, can constrain Shapiro time delay, r is magnitude and $s = \sin i$ is shape parameter.

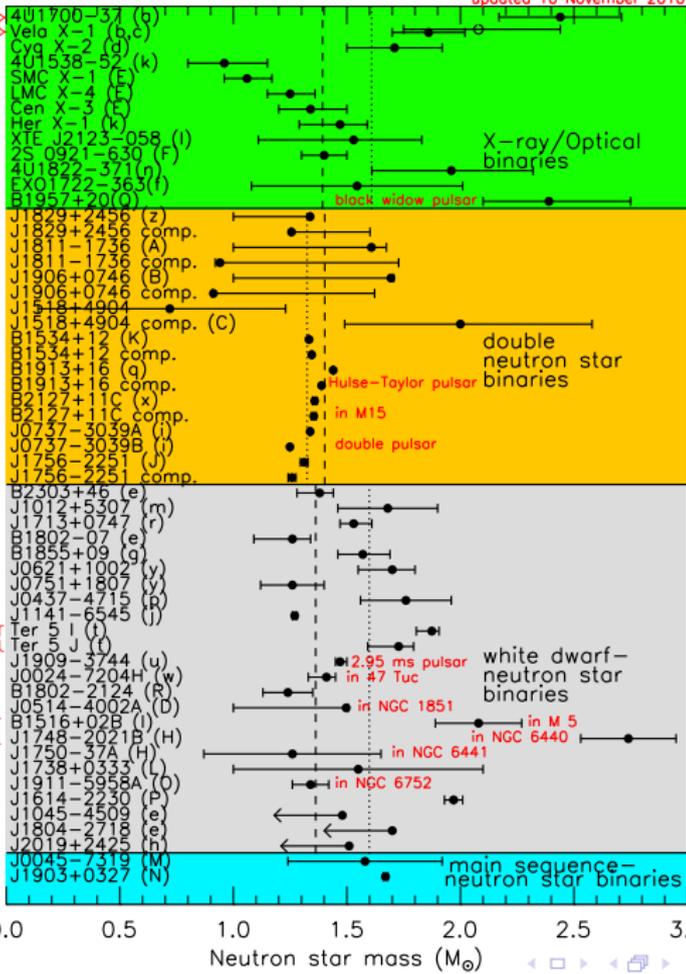


The Double Pulsar Binary



PSR0737-3039, Kramer et al. (2007)

Black hole? \Rightarrow
Firm lower mass limit? \Rightarrow



$M > 1.68 M_{\odot}$ {
95% confidence

Freire et al. 2007 {

Although simple average mass of w.d. companions is $0.27 M_{\odot}$ larger, weighted average is $0.08 M_{\odot}$ smaller

w.d. companion? statistics?

Demorest et al. 2010

Champion et al. 2008

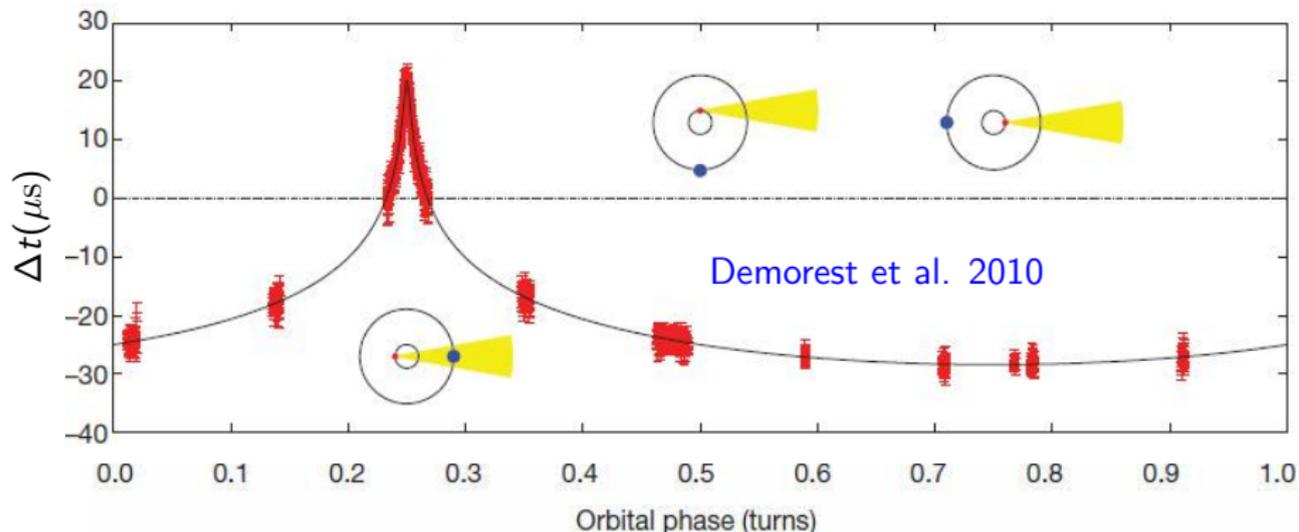
PSR J1614-2230

A 3.15 ms pulsar in an 8.69d orbit with an $0.5 M_{\odot}$ white dwarf companion.

Shapiro delay yields edge-on inclination: $\sin i = 0.99984$

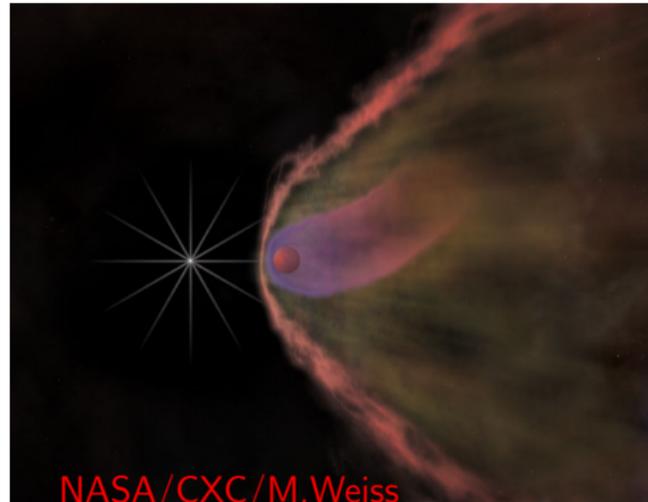
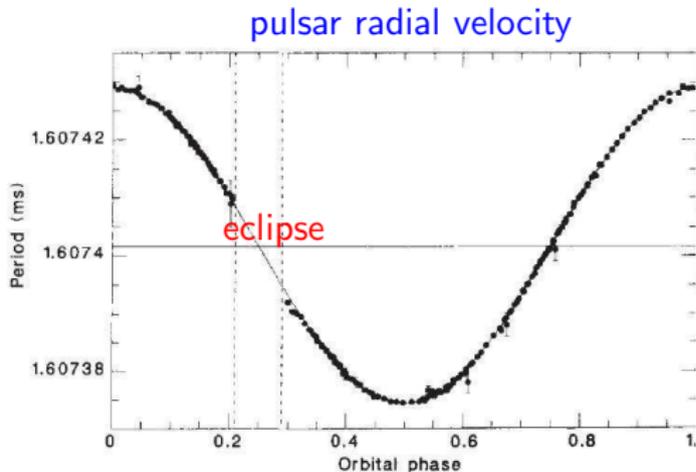
Pulsar mass is $1.97 \pm 0.04 M_{\odot}$

Distance > 1 kpc, $B \simeq \times 10^8$ G



Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $M_c \sim 0.03 M_\odot$ companion. Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe. It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe. Ablation by pulsar leads to eventual disappearance of companion. The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.



Implications of Maximum Masses

$$M_{max} > 2 M_{\odot}$$

- ▶ Upper limits to energy density, pressure and baryon density:
 - ▶ $\epsilon < 13.1\epsilon_s$
 - ▶ $p < 8.8\epsilon_s$
 - ▶ $n_B < 9.8n_s$
- ▶ Lower limit to spin period:
 $P > 0.56$ ms
- ▶ Lower limit to neutron star radius:
 $R > 8.5$ km
- ▶ Upper limits to energy density, pressure and baryon density in the case of a quark matter core:
 - ▶ $\epsilon < 7.7\epsilon_s$
 - ▶ $p < 2.0\epsilon_s$
 - ▶ $n_B < 6.9n_s$

$$M_{max} > 2.4 M_{\odot}$$

- ▶ Upper limits to energy density, pressure, baryon density:
 - ▶ $\epsilon < 8.9\epsilon_s$
 - ▶ $p < 5.9\epsilon_s$
 - ▶ $n_B < 6.6n_s$
- ▶ Lower limit to spin period:
 $P > 0.68$ ms
- ▶ Lower limit to neutron star radius:
 $R > 10.4$ km
- ▶ Upper limits to energy density, pressure, baryon density in the case of a quark matter core:
 - ▶ $\epsilon < 5.2\epsilon_s$
 - ▶ $p < 1.4\epsilon_s$
 - ▶ $n_B < 4.6n_s$

Neutron Star Matter Pressure and the Radius

$$p \approx Kn^\gamma$$

$$\gamma = d \ln p / d \ln n \sim 2$$

$$R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

$$R \propto p_f^{1/2} n_f^{-1} M^0$$

$$(1 < n_f/n_s < 2)$$

Wide variation:

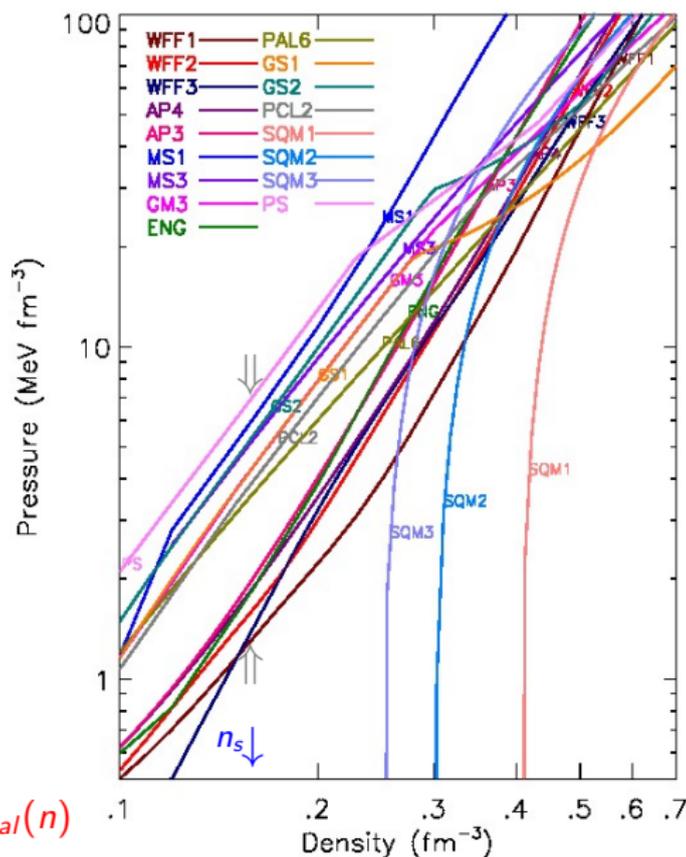
$$1.2 < \frac{p(n_s)}{\text{MeV fm}^{-3}} < 7$$

GR phenomenological result
(Lattimer & Prakash 2001)

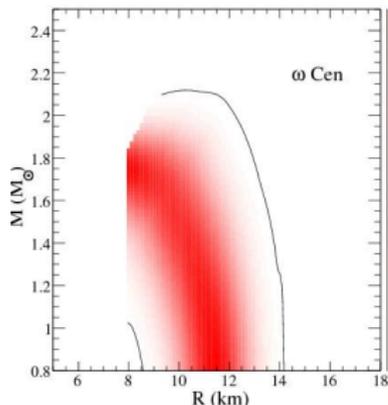
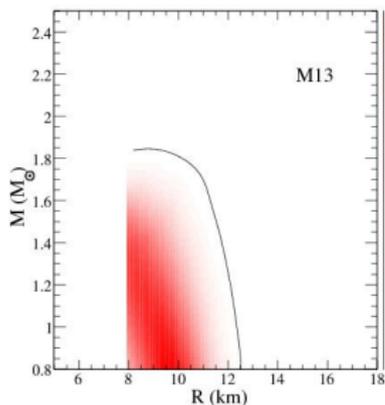
$$R \propto p_f^{1/4} n_f^{-1/2}$$

$$p_f = n^2 dE_{\text{sym}}/dn$$

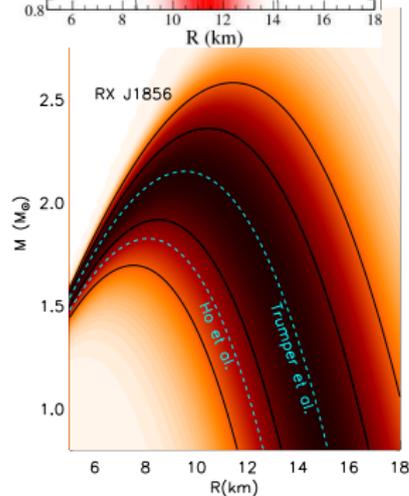
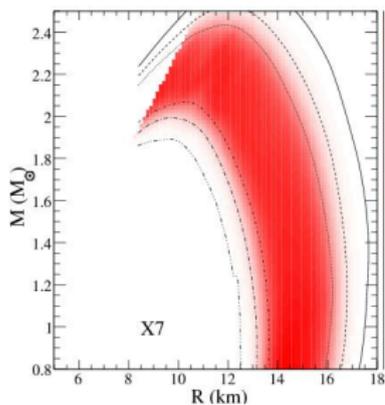
$$E_{\text{sym}}(n) = E_{\text{neutron}}(n) - E_{\text{symmetrical}}(n)$$



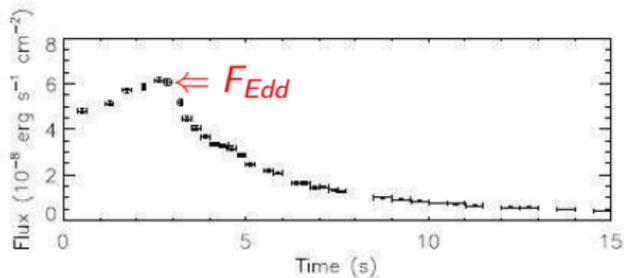
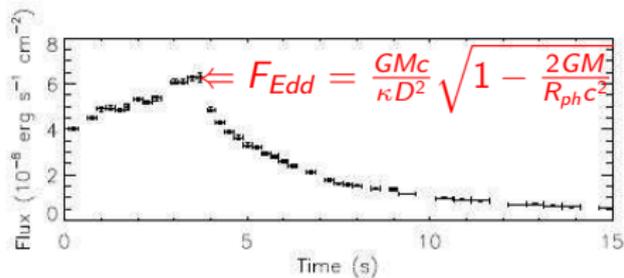
Inferred M-R Probability Estimates from Thermal Sources



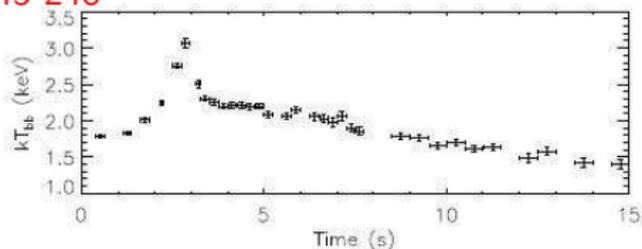
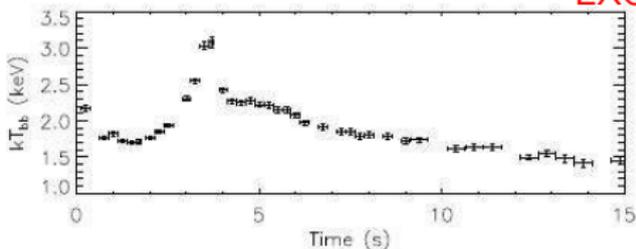
Steiner, Lattimer & Brown 2010



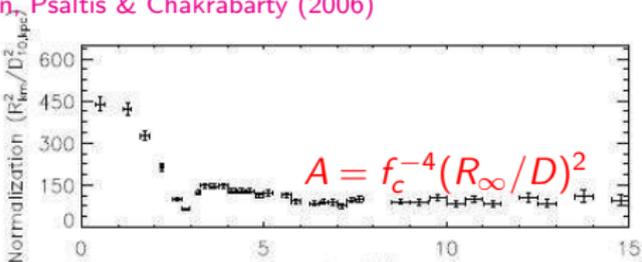
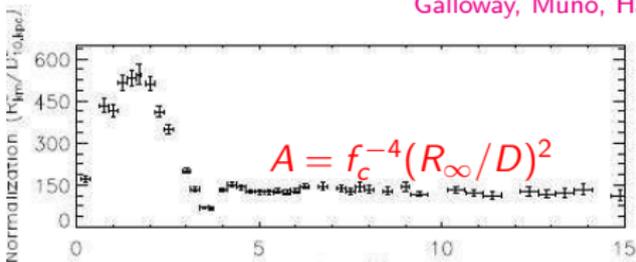
Photospheric Radius Expansion X-Ray Bursts



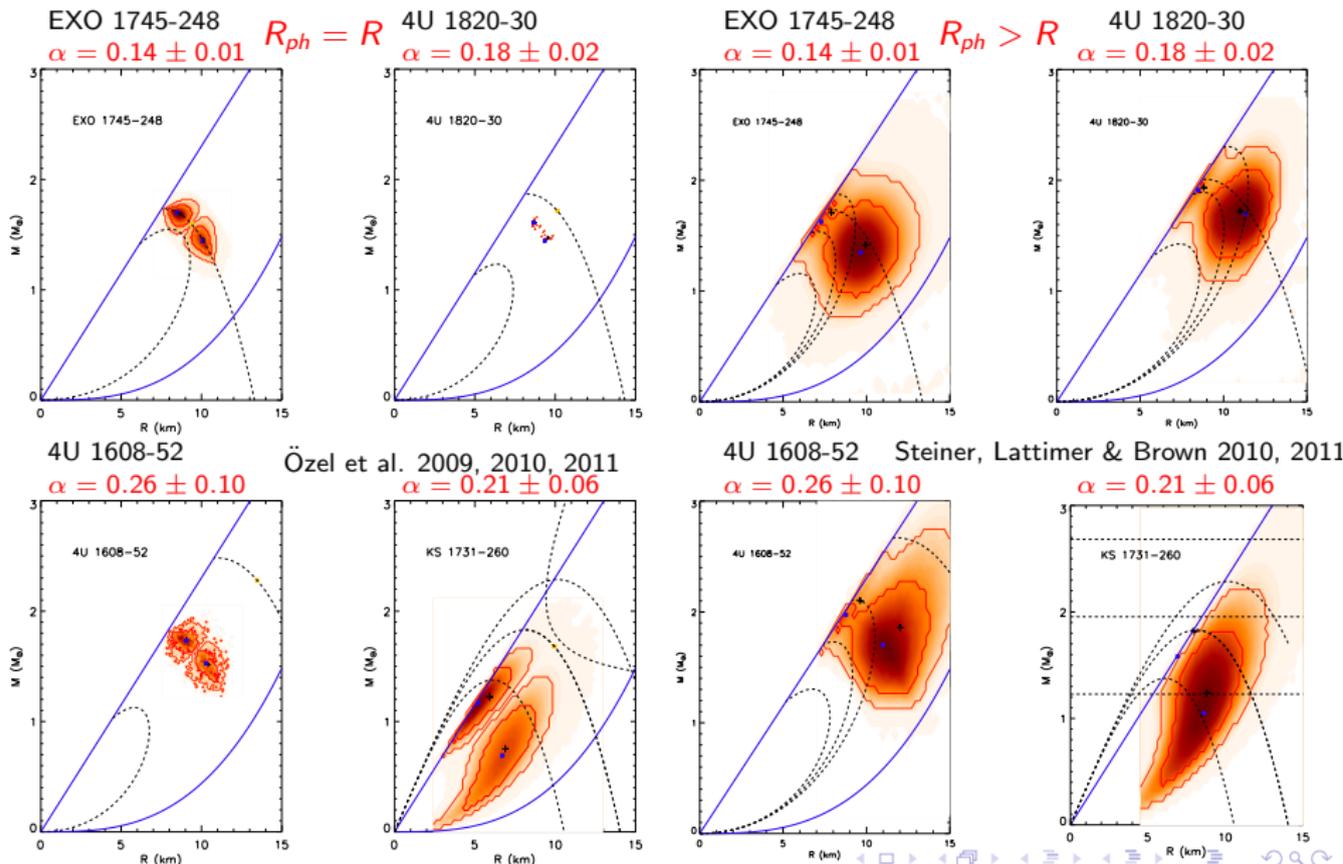
EXO 1745-248



Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)

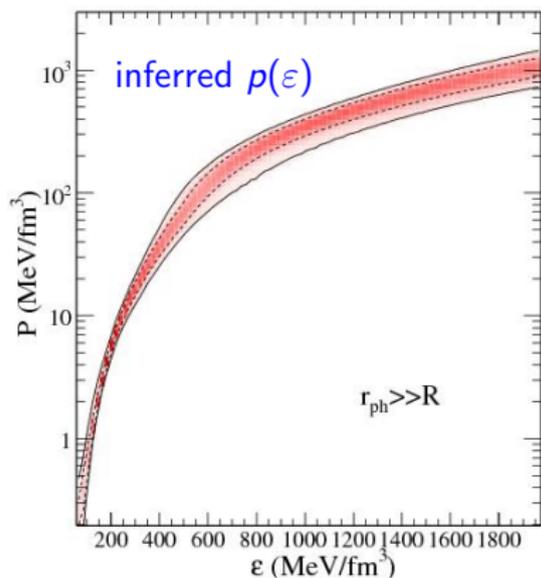


$M - R$ Probability Estimates from PRE Bursts

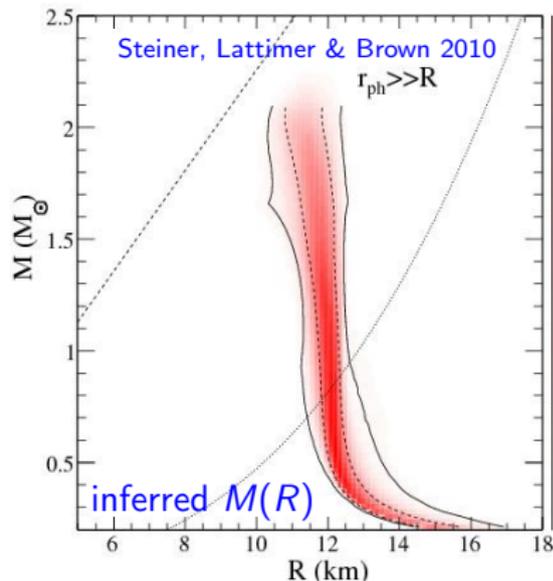


Bayesian TOV Inversion

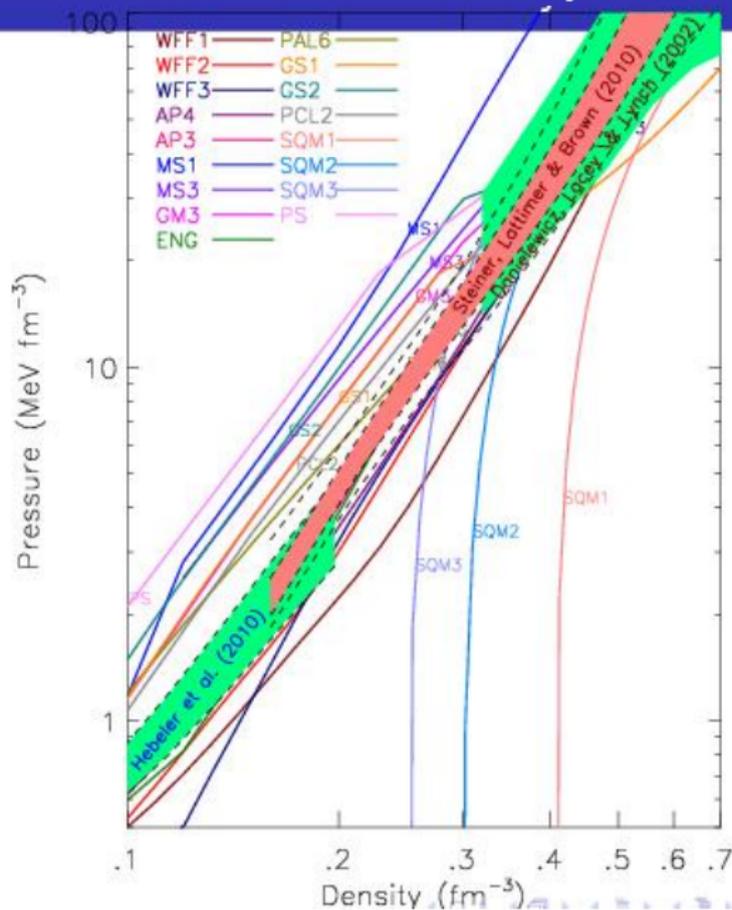
- ▶ $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- ▶ $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- ▶ $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1 ; $\varepsilon > \varepsilon_2$: Polytropic EOS with n_2



- ▶ EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- ▶ M and R probability distributions for 7 neutron stars treated equally.

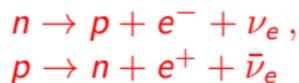


Consistency with Neutron Matter and Heavy-Ion Collisions



Urca Processes

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.



Energy conservation guaranteed by beta equilibrium

$$\mu_n - \mu_p = \mu_e$$

Momentum conservation requires

$$|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|.$$

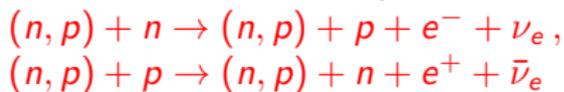
Charge neutrality requires $k_{Fp} = k_{Fe}$, therefore $|k_{Fp}| \geq 2|k_{Fn}|$.

Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.

With muons

$$(n > 2n_s), x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$$

If $x < x_{DU}$, bystander nucleons needed: modified Urca process.



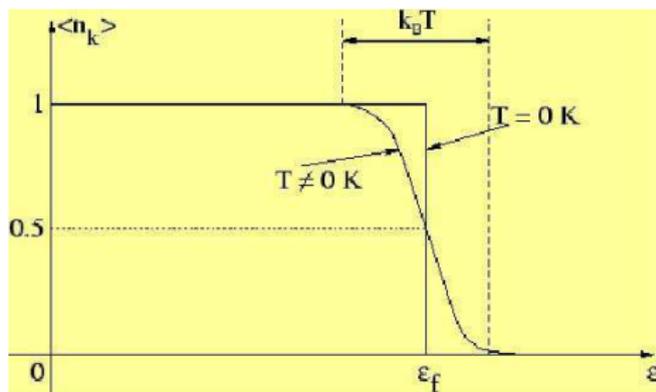
Neutrino emissivities:

$$\dot{\epsilon}_{MU} \simeq (T/\mu_n)^2 \dot{\epsilon}_{DU} \sim 10^{-6} \dot{\epsilon}_{DU}.$$

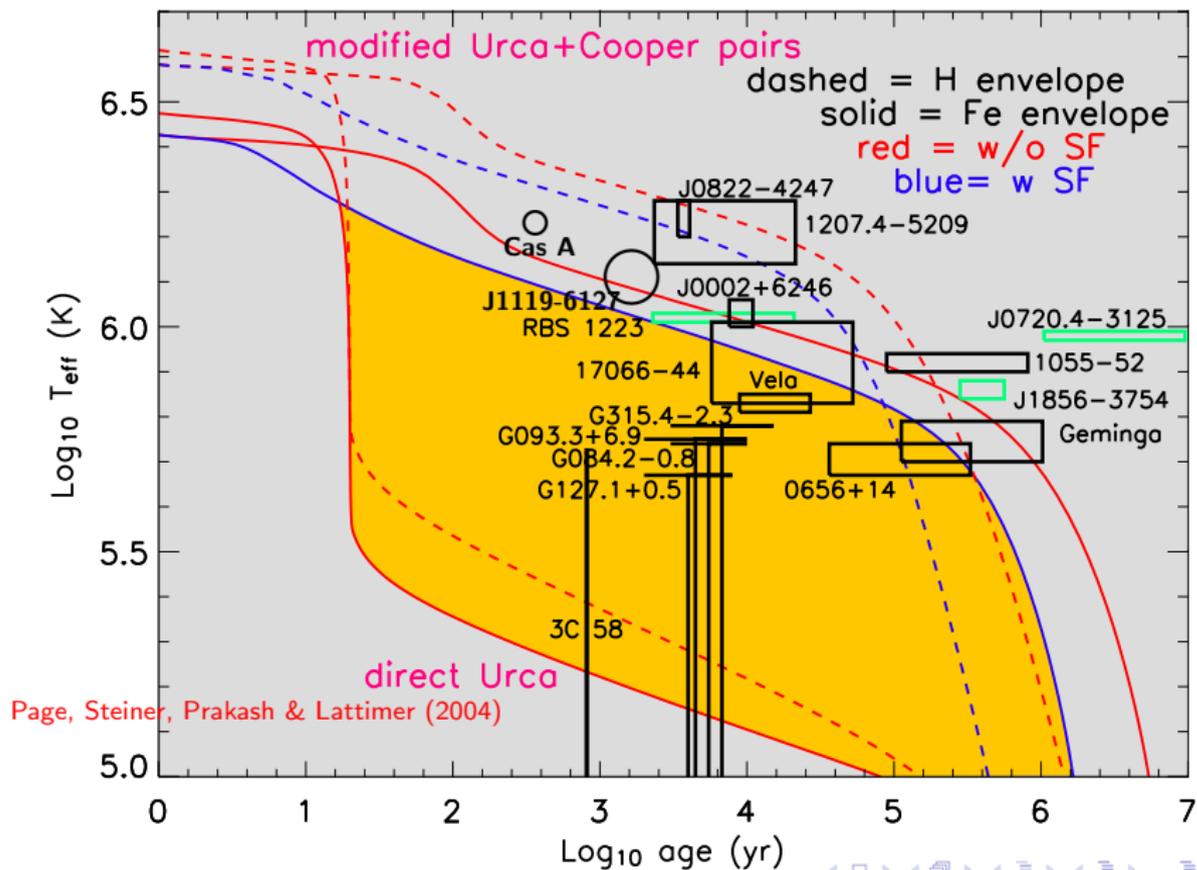
Beta equilibrium composition:

$$x_\beta \simeq (3\pi^2 n)^{-1} (4E_{sym}/\hbar c)^3$$

$$\simeq 0.04 (n/n_s)^{0.5-2}.$$



Neutron Star Cooling



Transitory Rapid Cooling

MU emissivity: $\dot{\epsilon}_{MU} \propto T^8$

PBF emissivity ($f \sim 10$):

$\dot{\epsilon}_{PBF} \propto F(T) T^7 \propto T^8 \simeq f \dot{\epsilon}_{MU}$

Specific heat: $C_V \propto T$

Neutrino dominated cooling:

$$C_V dT/dt = -L_\nu$$

$$\Rightarrow T \propto (t/\tau)^{-1/6}$$

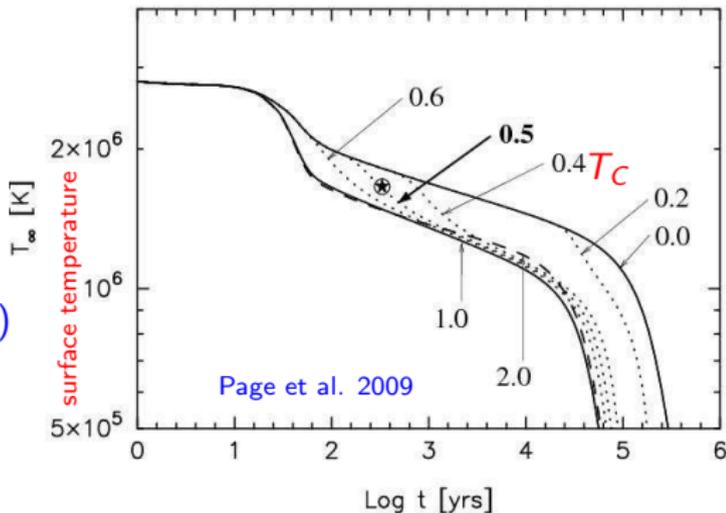
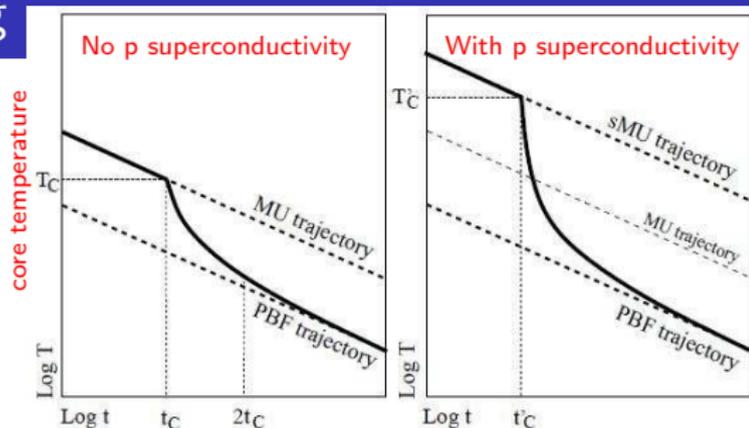
$$\tau_{PBF} = \tau_{MU}/f$$

$(d \ln T / d \ln t)_{transitory}$

$$\simeq (1 - 10)(d \ln T / d \ln t)_{MU}$$

$$\simeq (1 - 25)(d \ln T / d \ln t)_{MU} \text{ (p SC)}$$

Very sensitive to n^1S_0 critical temperature (T_C) and existence of proton superconductivity



Cas A

Remnant of Type IIb
(gravitational collapse,
no H envelope) SN in
1680 (Flamsteed).

3.4 kpc distance

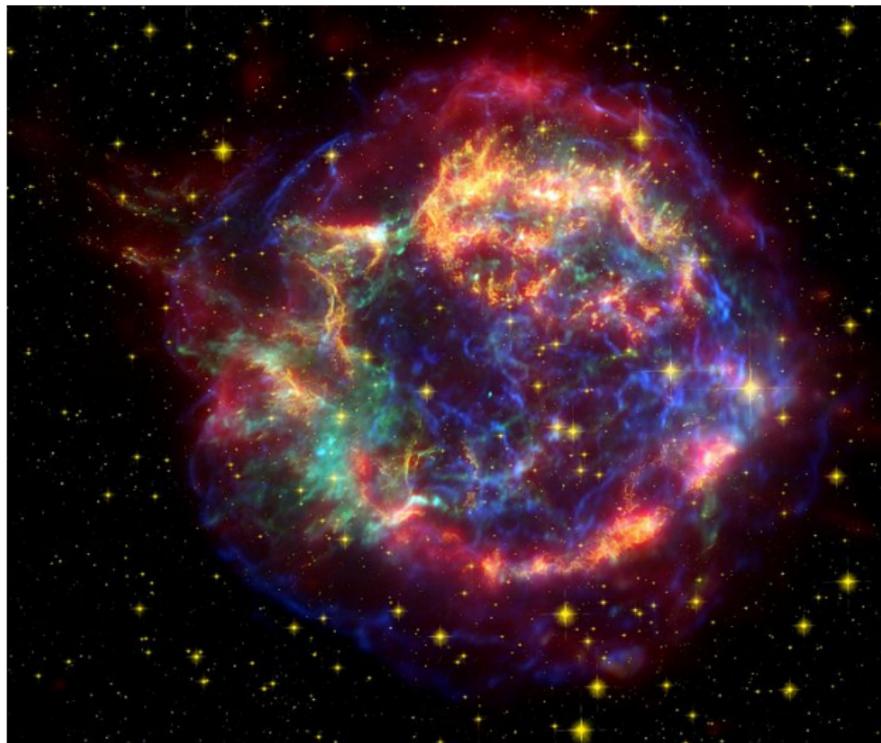
3.1 pc diameter

Strongest radio source
outside solar system,
discovered in 1947.

X-ray source detected
(Aerobee flight, 1965)

X-ray point source
detected
(Chandra, 1999)

1 of 2 known CO-rich
SNR (massive
progenitor and neutron star?)



Spitzer, Hubble, Chandra



Cas A Superfluidity

X-ray spectrum indicates thin C atmosphere,
 $T_e \sim 1.7 \times 10^8$ K
(Ho & Heinke 2009)

10 years of X-ray data show cooling at the rate
 $\frac{d \ln T_e}{d \ln t} = -1.23 \pm 0.14$
(Heinke & Ho 2010)

Modified Urca:
 $\left(\frac{d \ln T_e}{d \ln t}\right)_{MU} \simeq -0.08$

We infer that
 $T_C \simeq 5 \pm 1 \times 10^8$ K
 $T_C \propto (t_c L / C_V)^{-1/6}$

